Gauge-Invariant Approach to Meson Photoproduction Including the Final-State Interaction

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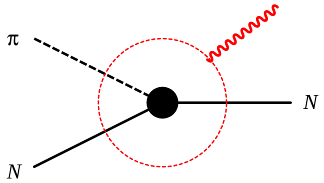
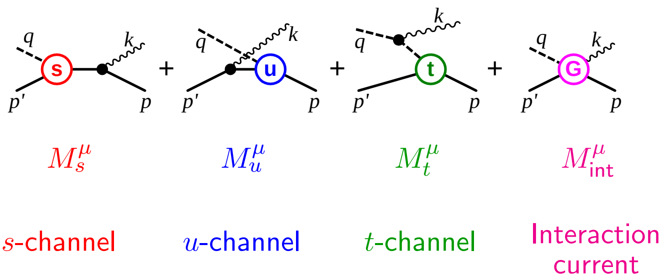
Example: $\gamma N \to \pi N$

Attaching a Photon to the πNN Vertex



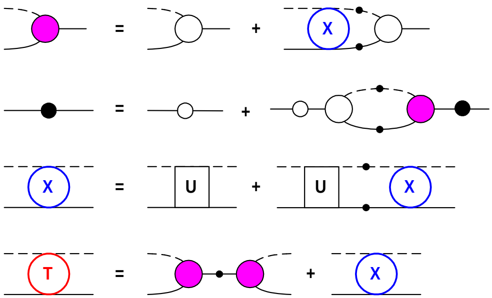


Total photoproduction amplitude

$$M^\mu = M_s^\mu + M_u^\mu + M_t^\mu + M_{\text{int}}^\mu$$

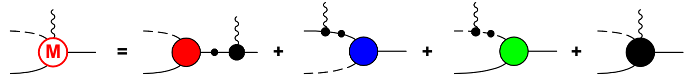
Pions, Nucleons, and Photons

Hadronic scattering: $\pi N \rightarrow \pi N$



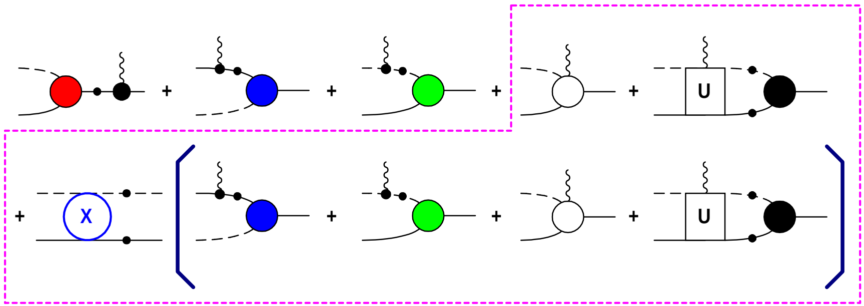
Photoproduction: $\gamma N \rightarrow \pi N$

Amplitude:



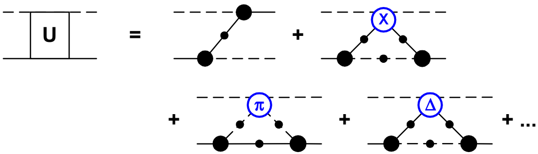
s-channel u-channel t-channel M^{μ}_{int}

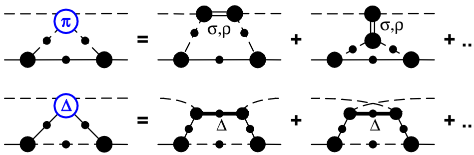
Theory: PRC **56**, 2041 (1997)



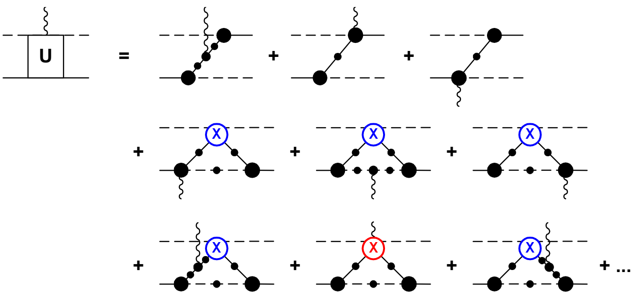
Interaction Current M^{μ}_{int}

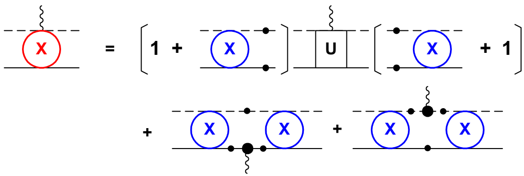
Hadronic Driving Terms



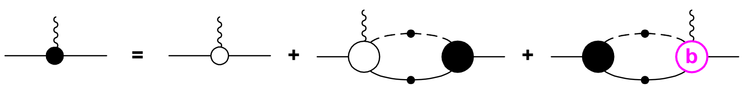


Electromagnetic Couplings to Driving Terms





Dressed Nucleon Current



This is VERY complicated!

- The full theory is gauge-invariant as a matter of course.
- In practice, truncations are necessary.
- Truncations usually destroy gauge invariance.

Prescriptions must be found that restore gauge invariance in practical applications.

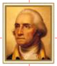
New Theoretical Tools for Nucleon Resonance Analysis

Old Theoretical Tool

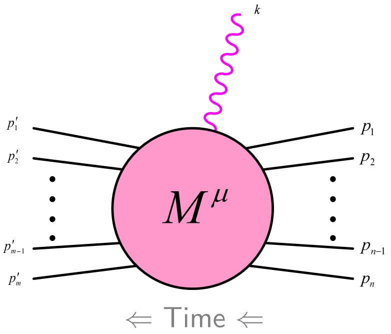
~~New Theoretical Tools~~ for Nucleon Resonance Analysis

Gauge Invariance

[1] J.C. Ward, Phys. Rev. **78**, 182 (1950)
[2] Y. Takahashi, Nuovo Cimento **6**, 371 (1957)
[3] E. Kazes, Nuovo Cimento **13**, 1226 (1959)



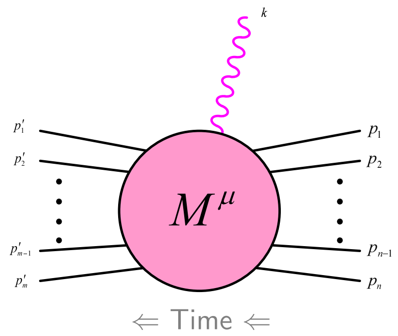
Gauge Invariance



Physical condition:

Current Conservation: $k_\mu M^\mu = 0$ all hadrons on-shell

Gauge Invariance



Physical condition:

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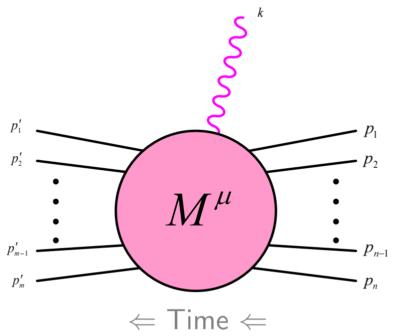
Recipe for Phenomenological Currents: $M^\mu = \tilde{M}^\mu - a^\mu \dfrac{k \cdot \tilde{M}}{k \cdot a}$ (a^μ arbitrary)

Gauge Invariance



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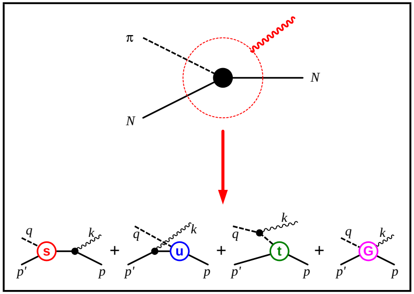
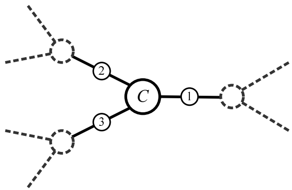
Not good enough for microscopic approaches!

Q: So why is it not okay to simply subtract terms to create a transverse current?

A: Because such currents are *always* transverse — on- and off-shell. This creates unacceptable *inconsistencies* in microscopic approaches.



To show this consider

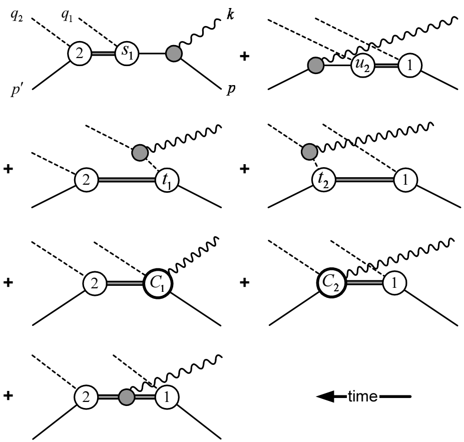


and attach photon in all places of the internal 3-point function...

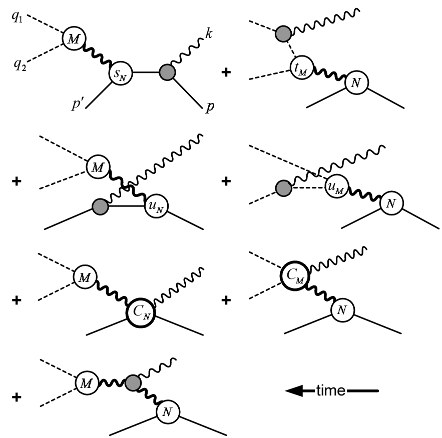
Details, details. . . $\longrightarrow$ Proof

Example: Two-pion production

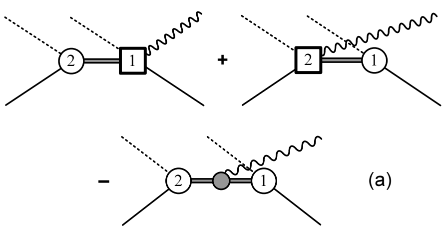
Basic Two-pion Production Mechanisms



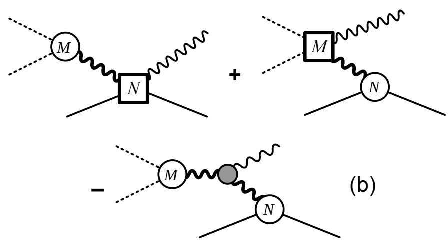
(a)



(b)

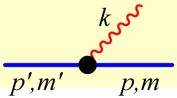


(a)



(b)

Ward–Takahashi Identities for the Nucleon and the Pion Currents

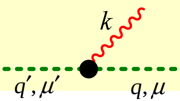


$$k_\mu \Gamma^\mu_N(p',p) = S_N^{-1}(p') Q_N - Q_N S_N^{-1}(p)$$ **nucleon**

Form factors: $f_n^{ij} = f_n^{ij}(p'^2, p^2; k^2)$

$$\Gamma^\nu_N(p',p) = \gamma^\nu Q_N + \left[t^\nu f_1^{00} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{00} \right] + \frac{\not p' - m}{2m} \left[t^\nu f_1^{10} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{10} \right]$$
$$+ \left[t^\nu f_1^{01} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{01} \right] \frac{\not p - m}{2m} + \frac{\not p' - m}{2m} \left[t^\nu f_1^{11} + \frac{i\sigma^{\nu\mu}k_\mu}{2m} f_2^{11} \right] \frac{\not p - m}{2m}$$

$t^\nu = \gamma^\nu k^2 - k^\nu \not k$

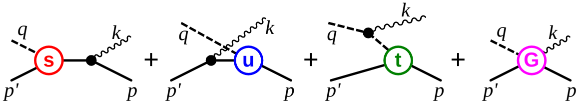


$$k_\mu \Gamma^\mu_\pi(q',q) = \Delta_\pi^{-1}(q') Q_\pi - Q_\pi \Delta_\pi^{-1}(q)$$ **pion**

$$\Gamma^\nu_\pi(q',q) = (q'+q)^\nu Q_\pi + \left[(q'+q)^\nu k^2 - k^\nu k \cdot (q'+q) \right] Q_\pi f(q'^2, q^2; k^2)$$

Generalized Ward–Takahashi Identity for the Pion-Production Current

$$k_\mu M^\mu = -[F_s\tau]S_{p+k}Q_N S_p^{-1} + S_{p'}^{-1}Q_N S_{p'-k}[F_u\tau] + \Delta^{-1}_{p-p'+k}Q_\pi\Delta_{p-p'}[F_t\tau]$$



Equivalently:

$$k_\mu M^\mu_{\text{int}} = -[F_s\tau]Q_N + Q_N[F_u\tau] + Q_\pi[F_t\tau]$$ ★★★★★

Note:

Charge conservation: $-\tau Q_N + Q_N\tau + Q_\pi\tau = 0$

Prescriptions to Restore Gauge Invariance

Gross/Riska

Absorb hadronic form factors in propagator and construct single-hadron currents to satisfy the appropriate WT identities.

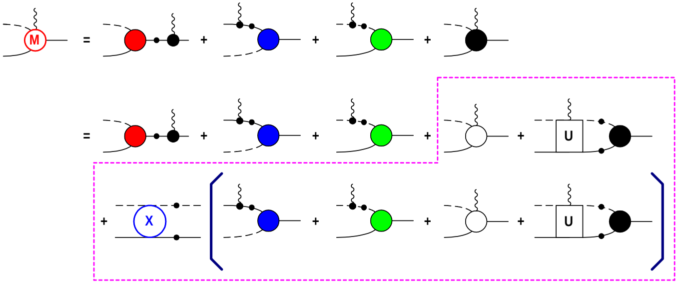
Ohta

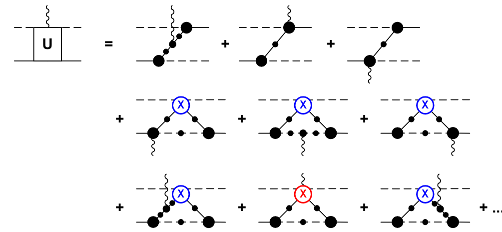
Construct contact current based on minimal substitution of πNN form factor. Basically removes hadronic structure from non-transverse contributions. Not applicable to explicit final-state interactions.

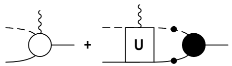
This work

Generalizes Ohta by introducing common form factor for non-transverse contributions only. Allows inclusion of explicit final-state interactions.

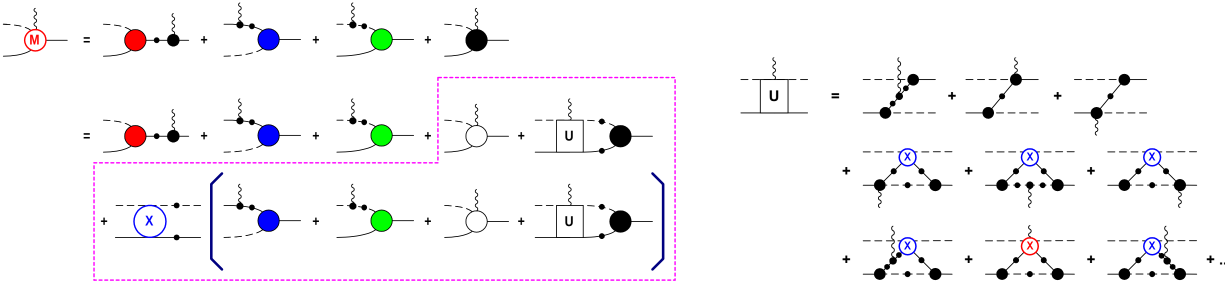
Lowest-Order Strategy

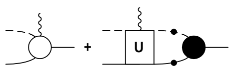




Replace  by phenomenological contact term such that WTI for interaction current is satisfied.

Lowest-Order Strategy



Replace  by phenomenological contact term such that WTI for interaction current is satisfied.

Result

$$M^\mu_{\rm int} = M^\mu_c + T^\mu + XG_0\Big[\underbrace{(M^\mu_u - m^\mu_u)}_{\text{transverse}} + \underbrace{(M^\mu_t - m^\mu_t)}_{\text{transverse}} + T^\mu\Big]$$

T^μ: undetermined transverse current

with

$$k_\mu M^\mu_c = -F_s e_i + F_u e_f + F_t e_\pi \ .$$

Phenomenological Choice for M_c^μ

$$M_c^\mu = g_\pi \gamma_5 \left\{ \left[\lambda + (1-\lambda) \frac{\not{q} - \beta \not{k}}{m' + m} \right] C^\mu - (1-\lambda) \frac{\gamma^\mu}{m'+m} \left[e_\pi f_t - \beta\, k_\rho C^\rho \right] \right\}$$

λ, β: free parameters

Non-singular auxiliary current:

$$C^\mu = -e_\pi \frac{(2q-k)^\mu}{t-q^2}(f_t - \hat{F}) - e_f \frac{(2p'-k)^\mu}{u-p'^2}(f_u - \hat{F}) - e_i \frac{(2p+k)^\mu}{s-p^2}(f_s - \hat{F}) \ ,$$

with

$$k_\mu C^\mu = e_\pi f_t + e_f f_u - e_i f_s$$

Subtraction function:

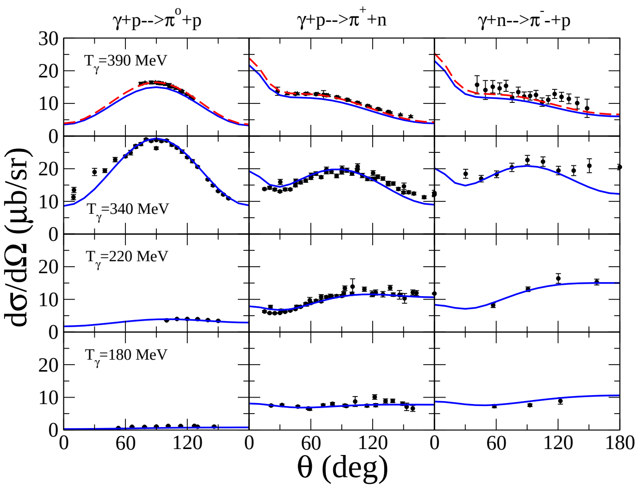
$$\hat{F} = 1 - \hat{h}\,(1-\delta_s f_s)(1-\delta_u f_u)(1-\delta_t f_t)$$

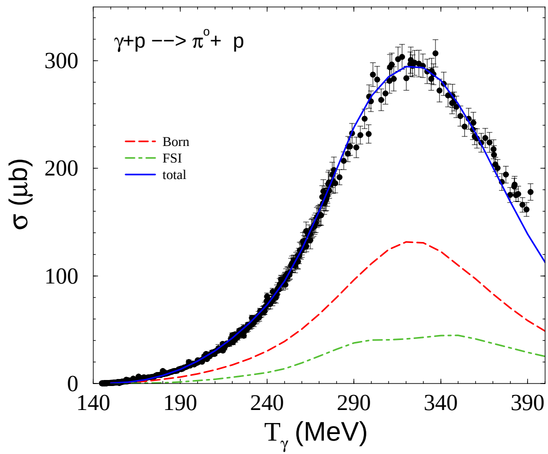
$\delta_x = 0, 1$

The function $\hat{h} = \hat{h}(s,u)$ is free fit function.

[Ohta: $\hat{h} = 0,\ X = 0,\ T^\mu = 0$]

First Application (straight out of the 'box' — not optimized)

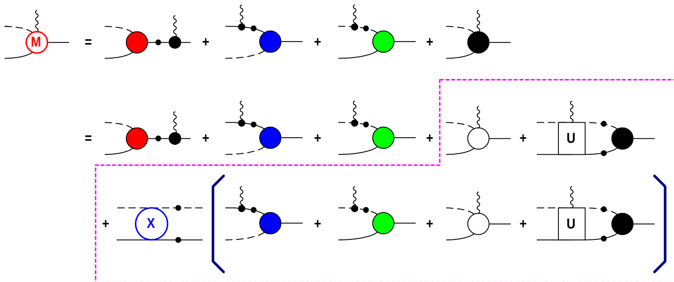


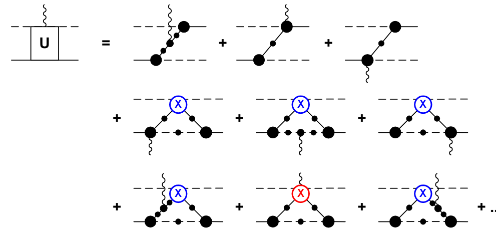


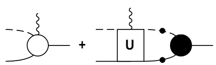
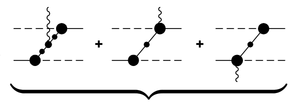
Includes

- ρ, ω, and a_1 exchanges in the t-channel;
- Δ in s- and u-chanels;
- FSI with Jülich πN T-matrix.

Next Order

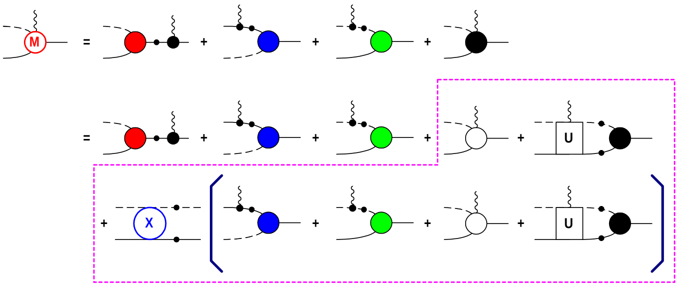


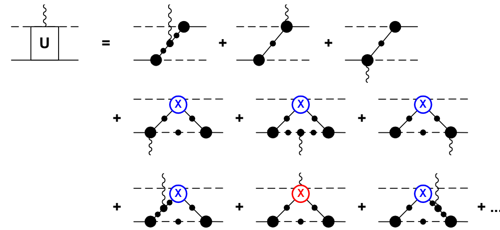


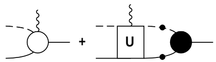
Instead of replacing all of  , take into account  explicitly.

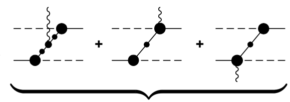
$=E^{\mu}$ exchange current

Next Order





Instead of replacing all of 

, take into account  explicitly.

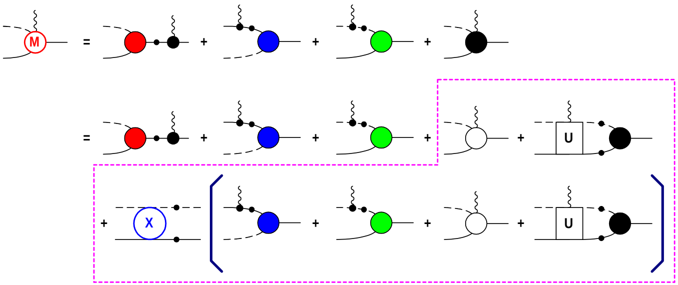
$=E^\mu$ exchange current

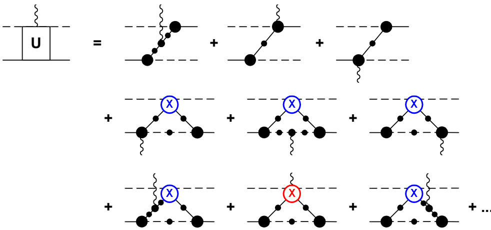
General gauge-invariance condition for U^μ

$$k_\mu U^\mu(p',q',p,q) = Q'_\pi U(p',q'-k,p,q) + Q'_N U(p'-k,q',p,q) - U(p',q',p,q+k)Q_\pi - U(p',q',p+k,q)Q_N$$

Holds also true for every subset of U^μ that originates from attaching a photon in all possible ways to a single two-body irreducible hadron graph $\Longrightarrow$ also true for E^μ

Next-Order Result





Previously

$$M^\mu_{\rm int} = M^\mu_c + T^\mu + X G_0 \Big[\underbrace{(M^\mu_u - m^\mu_u)}_{\text{transverse}} + \underbrace{(M^\mu_t - m^\mu_t)}_{\text{transverse}} + T^\mu \Big]$$

T^μ: undetermined transverse current

Now: T^μ is replaced by

$$T^\mu = \left[E^\mu - \tilde{E}^\mu \right] G_0 [F\tau] + T'^\mu$$

T'^μ: undetermined transverse current

Phenomenological Subtraction Current

$$\tilde{E}^\mu = g_\pi^2 \gamma_5 \left[\lambda - (1-\lambda)\frac{\not{q}}{m'+m_N}\right] S_N \gamma_5 \left[\lambda + (1-\lambda)\frac{\not{q}'}{m+m_N}\right] D^\mu$$

$$+ (1-\lambda) g_\pi^2 f_\pi f_1 e_\pi \frac{\gamma_5\gamma^\mu}{m'+m_N} S_N \gamma_5 \left[\lambda + (1-\lambda)\frac{\not{q}'}{m+m_N}\right]$$

$$- (1-\lambda) g_\pi^2 f_2 f_{\pi'} \gamma_5 \left[\lambda - (1-\lambda)\frac{\not{q}}{m'+m_N}\right] S_N e'_\pi \frac{\gamma_5\gamma^\mu}{m+m_N}$$

Non-singular auxiliary current:

$$D^\mu = e'_\pi \frac{(2q'-k)^\mu}{(q'-k)^2-q'^2}(f_2 f_{\pi'} - \hat{G}) + e'_N \frac{(2p'-k)^\mu}{(p'-k)^2-p'^2}(f_{N'} f_1 - \hat{G})$$

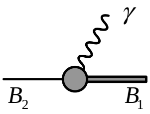
$$+ e_\pi \frac{(2q+k)^\mu}{(q+k)^2-q^2}(f_\pi f_1 - \hat{G}) + e_N \frac{(2p+k)^\mu}{(p+k)^2-p^2}(f_2 f_N - \hat{G}) \;,$$

with $\hat{G} = 1 - \hat{g}\,\big(1-f_\pi f_1\big)\big(1-f_2 f_N\big)\big(1-f_2 f_{\pi'}\big)\big(1-f_{N'} f_1\big)$

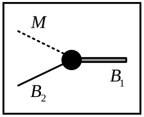
$\hat{g}$: free fit function

Note: $k_\mu D^\mu = e_\pi f_\pi f_1 + e_N f_2 f_N - e'_\pi f_2 f_{\pi'} - e'_N f_{N'} f_1$

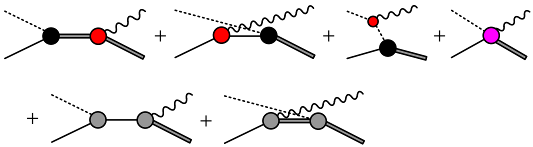
Transition Currents



- Gauge invariance demands that transition currents be transverse.
  Outline of proof:

  Coupling the photon to MB_1B_2 vertex, 

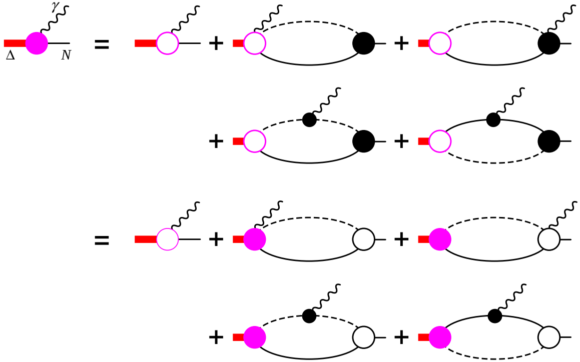
, produces



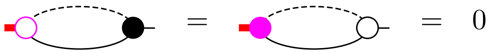
  First line produces correct off-shell relation. Terms in second line must vanish individually. QED

- In a consistent microscopic approach, this should be ensured dynamically. There should be no need to adjust the transversality manually.

Example: $\gamma N \to \Delta$



Transversality of this current is ensured by the fact that $N \to \Delta$ is not possible as a physical process, i.e.



With consistent dynamics, this current satisfies

$$\Gamma^{\beta\mu} = G_1\,\gamma_5\left(k^\beta\gamma^\mu - g^{\beta\mu}\,\not{k}\right) + G_2\,\gamma_5\left(k^\beta P^\mu - g^{\beta\mu}\,k\cdot P\right) + G_3\,\gamma_5\left(k^\beta k^\mu - g^{\beta\mu}\,k^2\right)$$

as a matter of course, where $P = (p+p')/2 = (2p+k)/2$. There is no need for subtractions.

Summary

- Gauge invariance is a fundamental symmetry — without it results become arbitrary.
- Current conservation $\boxed{k_\mu M^\mu = 0}$ necessary, but not sufficient, for gauge invariance of microscopic theories.
- Subtractions $M^\mu = \tilde{M}^\mu - a^\mu \dfrac{k\cdot\tilde{M}}{k\cdot a}$ in general do not ensure gauge invariance.
- Most of the popular dynamical models are not gauge-invariant; despite claims to the contrary.
- Generalized Ward–Takahashi identities are necessary and sufficient.
- Real-world calculations require truncations of dynamical mechanisms that destroy gauge invariance.
- Prescriptions required to restore gauge invariance.
- Transition currents are transverse and this property must be ensured dynamically.
- Formalism allows inclusion of final-state interaction.
- FSI dynamics can be refined step-by-step in controlled manner.