Models for Extracting N^* Parameters from Meson-Baryon Reactions

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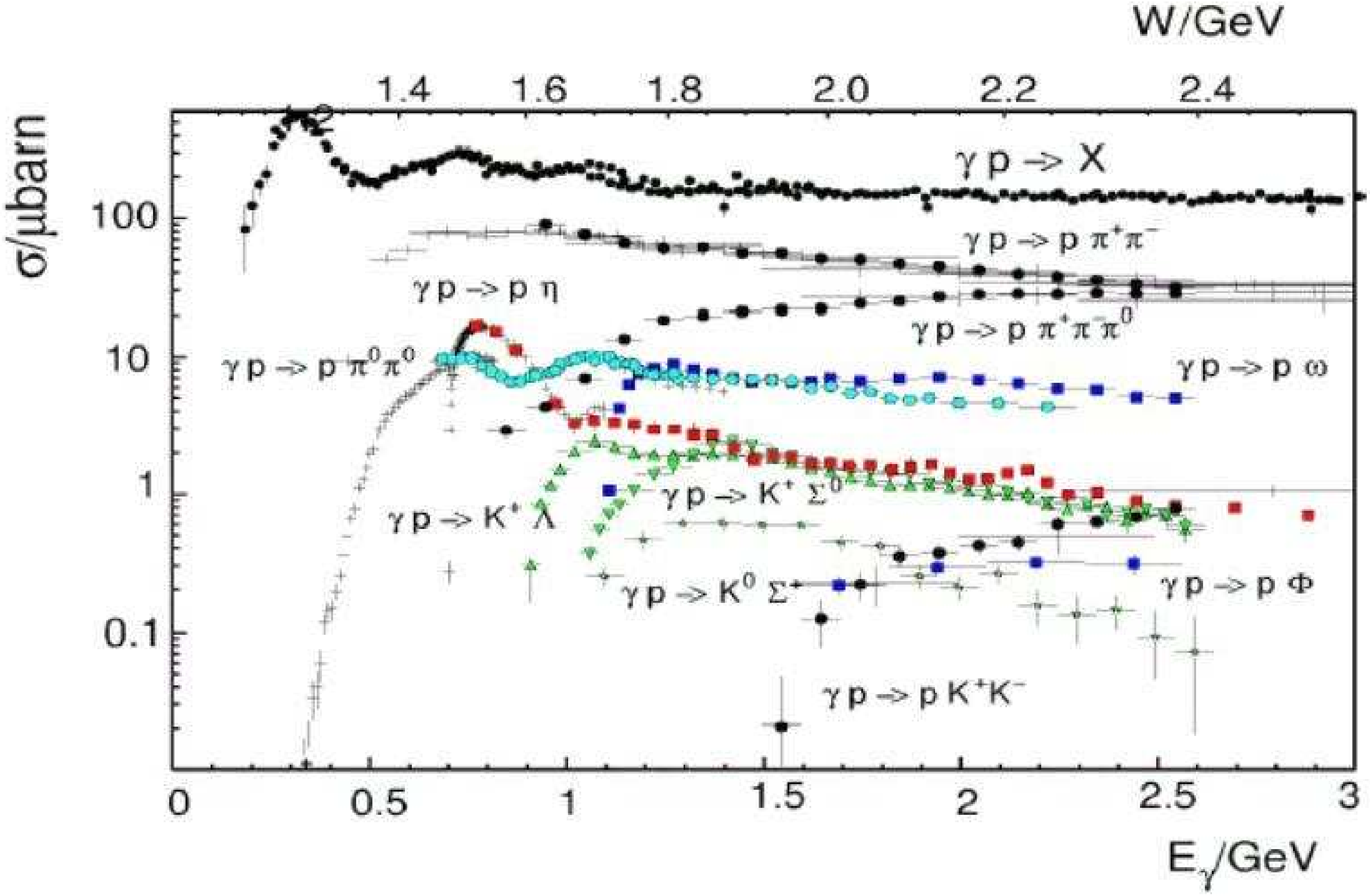
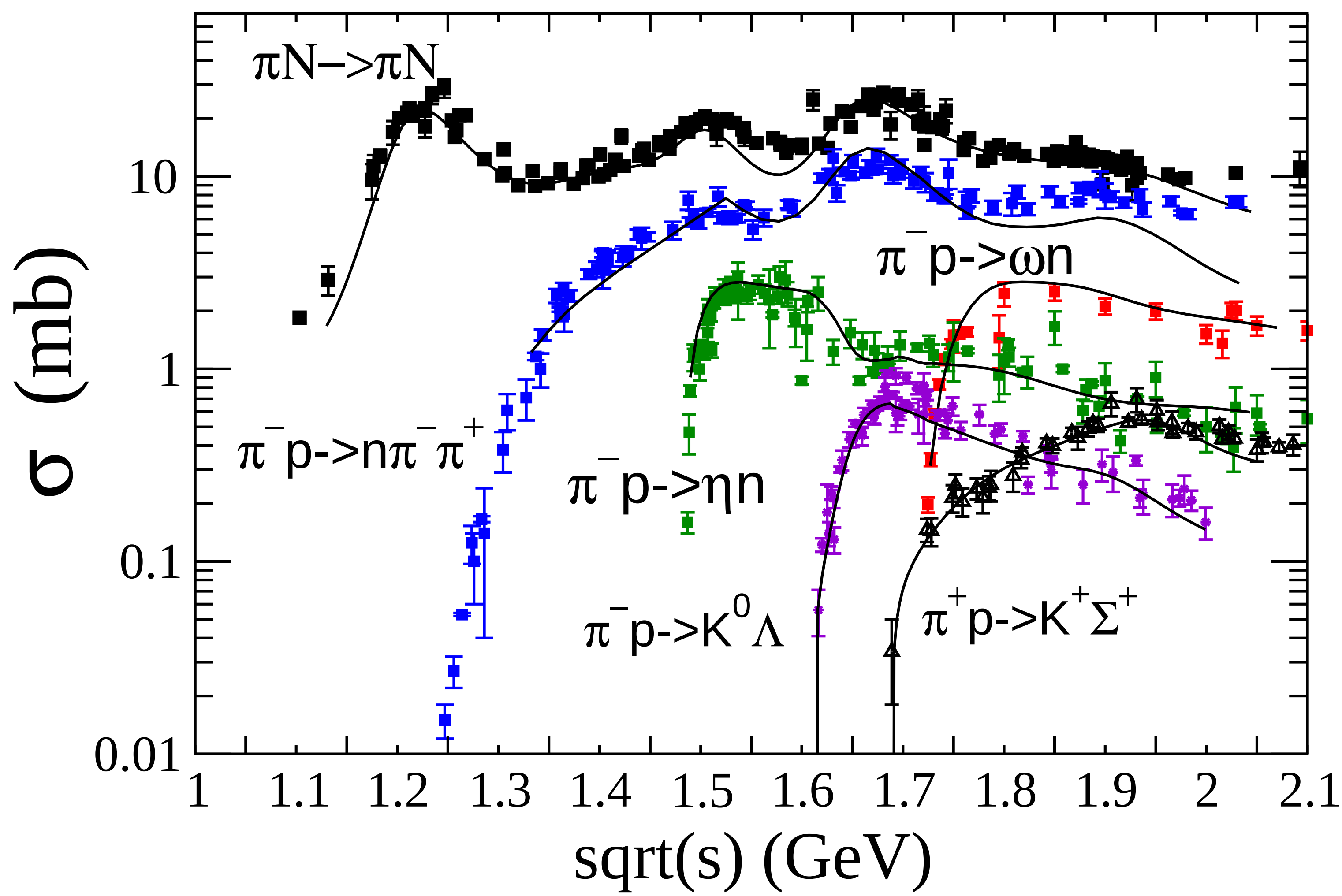


Data of γp reaction cross sections
(from Ostrick and Schmieden)



Data of πN reaction cross sections
(from V. Shklyar)

Challenge :

Extensive data of electromagnetic production of
π, η, K, ω, ϕ, and $\pi\pi N$ (ρN, $\pi\Delta$)

$\downarrow\downarrow$

Extract properties of nucleon resonances (N^*)

$\downarrow\downarrow$

Understand non-perturbative QCD :

- Confinement of constituent quarks
- Chiral dynamics of meson cloud of baryons

Tasks :

- Perform Amplitude Analyses of data

  $\rightarrow$

  Extract N^* parameters

- Develop Dynamical Reaction Models

  $\rightarrow$

  Interpret N^* parameters in terms of QCD :

  - Hadron Models (now)
  - Lattice QCD (near future)

Status :

- In the Δ region
  - Well developed
  - Amplitude Analyses and Dynamical Reaction Models are complementary

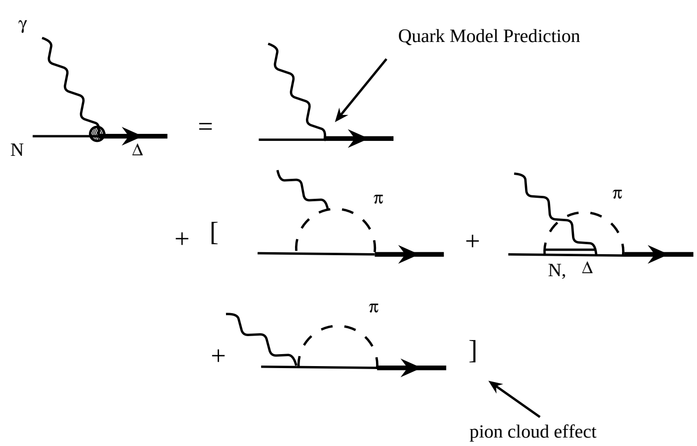
Example: $\gamma N \rightarrow \Delta$ M1 transition

- All amplitude analyses : $G_M(0) = 3.50 \pm 0.2$
- Disagree with the quark model : $\frac{G_M^{Exp.}(0)}{G_M^{Q.M.}(0)} \sim 1.4$

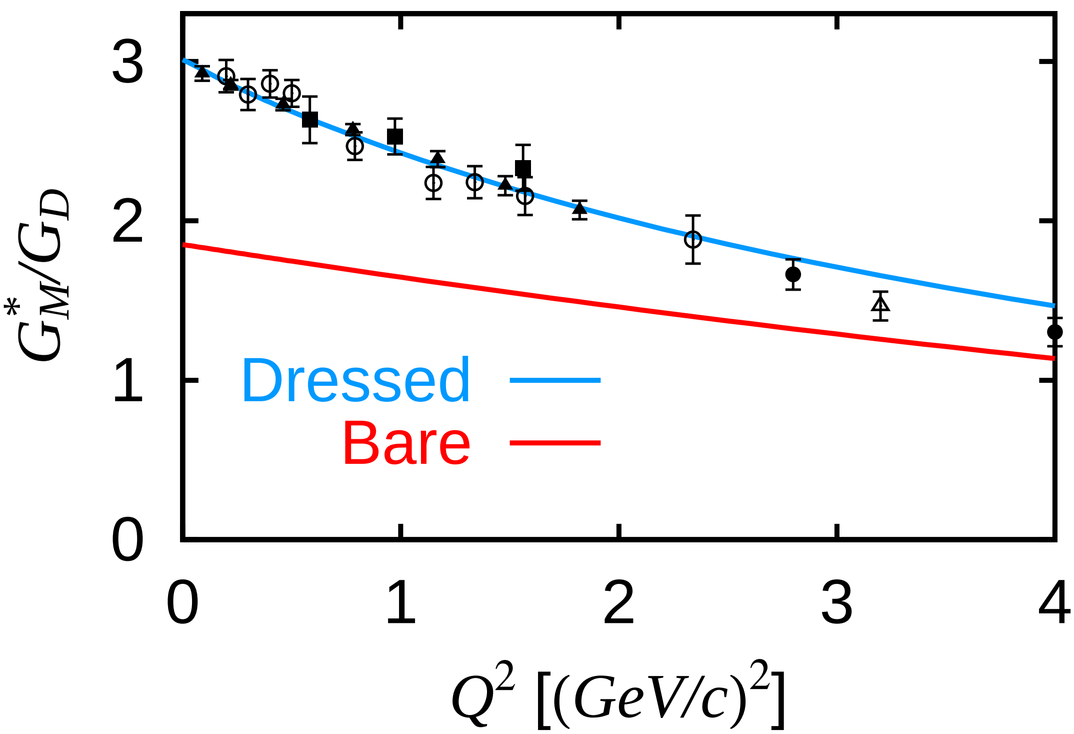
Solution: Develop Dynamical Models

$\rightarrow$

Find: due to meson cloud



$\gamma N \to \Delta$ Magnetic Dipole $\quad G_M(Q^2)$



- Pion cloud has a very large effect on G_M

Recent progress in the Δ region :

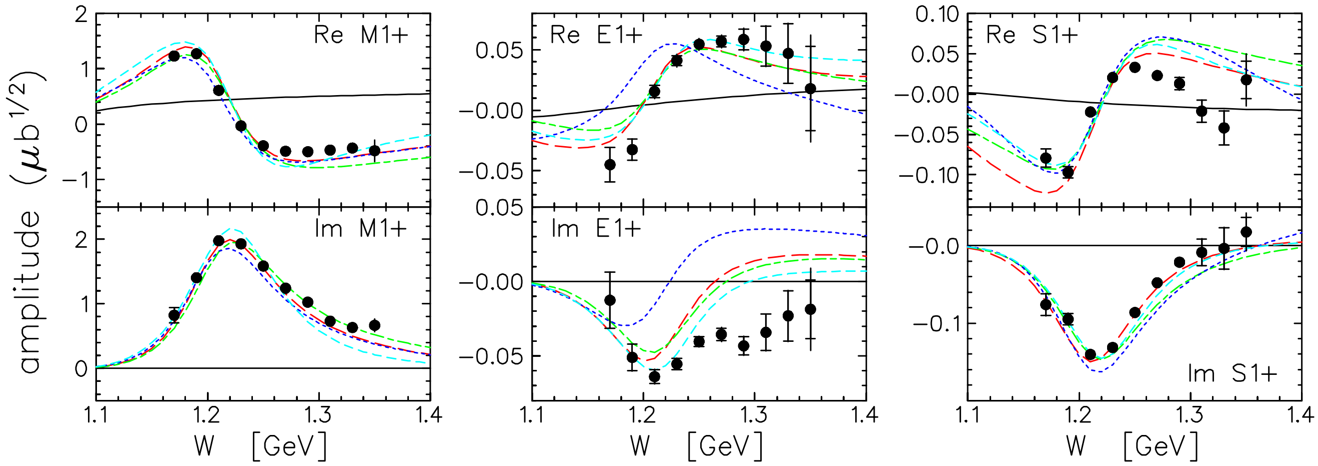
- 16 reponse functions of $p(\vec{e}, e\vec{p})$ at $Q^2 = 1$ (GeV/c)^2 have been obtained at JLab
  (allow almost model independent amplitude analysis)

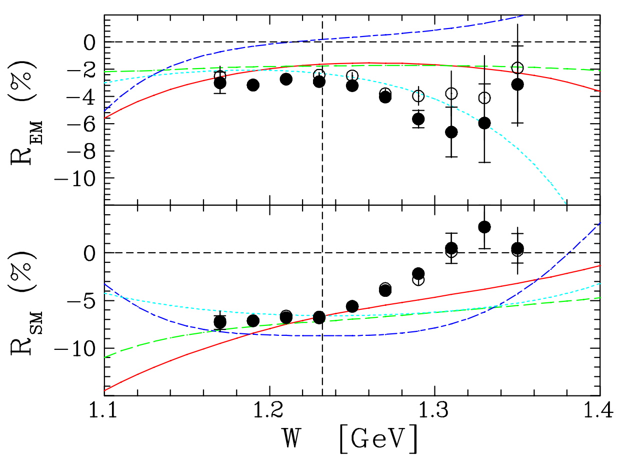
- LQCD calculations of N-Δ form factors are available

- Low Q^2 data have been obtained at JLab, Mainz, MIT-Bates
  (reveal Q^2-evolution of meson cloud effects on N-Δ)

- Data of $\vec{d}(\vec{\gamma}, \pi N)N$ have been obtained at LEGS
  (will provide $\gamma n \rightarrow \pi N$ multipoles :A. Sandorfi's talk)

Model Independent Amplitudes at $Q^2 = 1$ (GeV/c)^2
(J. Kelly et al. (2005))

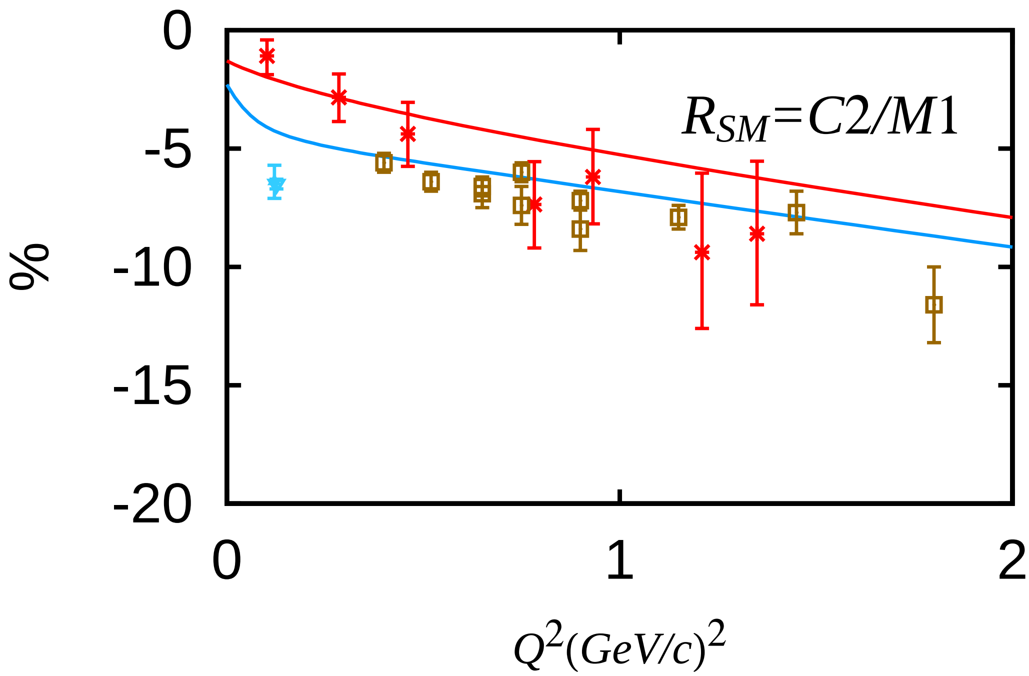




Curves : SL, DMT, MAID, SAID

Recent Results from LQCD

R_{SM} : measure deformation of N or Δ



red-bar : LQCD of C. Alexandrou et al. (2005)

red curve : Bare f.f of SL Model

blue curve : Dressed f.f. of SL Model

- In the second and third resonance regions:

  Open channels: ηN, $\pi\pi N$ ($\pi\Delta$, ρN), ωN, KN $\cdots$

  $\rightarrow$

  Need to develop coupled-channel approaches

  This talk : Review current developments

Outline

- Introduce a reaction formulation :
  - derive and compare
    current models of meson production reactions
- Review the status of the coupled-channel analyses
- Concluding Remarks

Reaction Theories

- Based on Hamiltonian or Bethe-Salpeter Equations :

$$T(E) = V + V \frac{1}{E - H_0 + i\epsilon} T(E)$$

$$V = interactions$$

Can be used to derive

- Unitary Isobar Models :
  MAID
  Jlab/Yerevan UIM
- Multi-channel K-matrix models :
  SAID
  Giessen model, KVI model
  Kent State University (KSU)
- Carnegie-Mellon Berkeley (CMB) Model
- Dynamical Reaction models
  Juelich, SL, DMT, Ohio-Utrecht $\cdots$
  Chiral SU(3) models

- Based on Dispersion Relations :

$$Re A^I(s,t) = B^I + \frac{1}{\pi} P \int_{s_{thr}}^{\infty} [\frac{1}{s'-s} + \frac{\epsilon^I \xi_i}{s'-u}] Im A^I(s',t)$$

Recent works :

- Constraint on πN amplitudes in SAID
  R.Arndt, I. Strakovsky, R. Workman
  and collaborators (1996, 2004)

- $\gamma N \to \pi N$
  O. Hanstein, D. Drechsel, and L. Tiator (1998)

- $\gamma N \to \pi N, \eta N$
  I. Aznauryan (1998, 2003)

Will not be covered in this talk

Derivations of Models

- Define K operator:

$$T(E) \;=\; V + V\frac{1}{E - H_0 + i\epsilon}T(E)$$

$\rightarrow$

$$T(E) \;=\; V + V[\frac{P}{E - H_0} - i\pi\delta(E - H_0)]T(E)$$

$$K(E) \;=\; V + V\frac{P}{E - H_0}K(E)$$

P : the principal-value integration.

$\rightarrow$

$$T(E) = K(E) - T(E)[i\pi\delta(E - H_0)]K(E)$$

$\rightarrow$

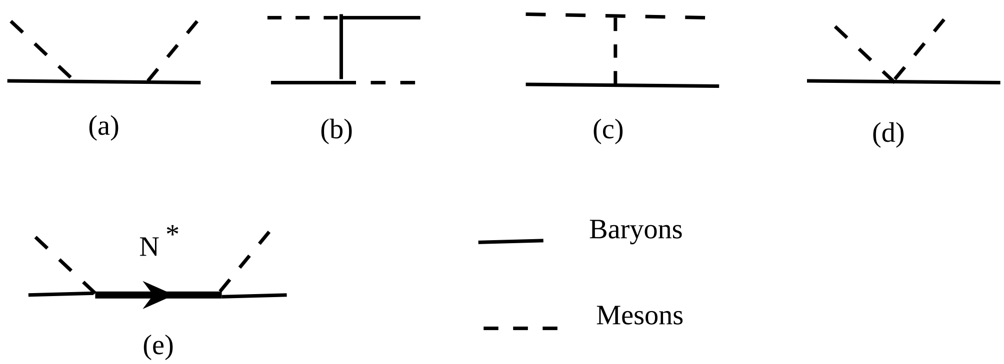
Lead to on-shell relations between T and K

- Define interactions :

$$V = v^{bg} + v^{R}$$

Non-resonant term: v^{bg}

Resonant term: $v^{R} = \frac{\Gamma_i^{\dagger}\Gamma_i}{E - M_{N_i^*}}$



Approaches :

- Start with $V = v^{bg} + v^{R}$:

$$T_{a,b}(k_a, k_b, E) = V_{a,b}(k_a, k_b) + \sum_c \int dk \frac{V_{a,c}(k_a, k) T_{c,b}(k, k_b)}{E - E_{M_c}(k) - E_{B_c}(k) + i\epsilon}$$

a,b $= \pi N,\ \gamma N,\ \eta N,\ \omega N,\ KY,\ \rho N,\ \pi\Delta\ (\pi\pi N)$

  – Need off-shell information
  – Equations for Dynamical Models

- Start with K matrix:

A matrix relation:

$$T_{a,b}(E) = \sum_c [(1 + iK(E))^{-1}]_{a,c} K_{c,b}(E)$$

a,b $= \pi N,\ \gamma N,\ \eta N,\ \omega N,\ KY,\ \rho N,\ \pi\Delta\ (\pi\pi N)$

  – Need only on-shell information
  – Equations for K-matrix Models

Derivations

- Unitary Isobar Model (UIM) :
  - start with K matrix
  - channels : γN, πN (or ηN)

$\rightarrow$

$\gamma N \rightarrow \pi N$ amplitude :

$$T_{\pi N,\gamma N} = \frac{1}{1+iK_{\pi N,\pi N}(E)} K_{\pi N,\gamma N}(E)$$
$$= e^{i\delta_{\pi N}} cos\delta_{\pi N} K_{\pi N,\gamma N}(E)$$
$$\sim e^{i\delta_{\pi N}} cos\delta_{\pi N} V_{\pi N,\gamma N}$$

$V_{\pi N,\gamma N} =$ Tree-diagrams

$\delta_{\pi N}$: πN phase shifts

$\rightarrow$

Satisfy Watson Theorem in $W < 1.3$ GeV

– Mainz and Jlab/Yerevan UIM :

1. Include of N^* by using Walker's parameterization
2. Unitarize the total amplitude

$\rightarrow$

$$T_{\pi N,\gamma N}(UIM) = e^{\delta} cos\delta[v^{bg}_{\pi N,\gamma N}] + \sum_{N_i^*} T^{N_i^*}_{\pi N,\gamma N}(W)$$

$$T^{N_i^*}_{\pi N,\gamma N}(E) = f_{\pi N}(W) \frac{\Gamma^{tot} M_i e^{i\Phi_i}}{M_i^2 - W^2 - iM_i\Gamma^{tot}} A_{\gamma N}(W)$$

Φ_i : Unitarization Phase

Results from MAID and JLab/Yerevan UIM :

1. Successful in extracting Δ parameters
2. Can fit pion production data up to W = 2 GeV.

Comments :

Coupled-channel effects are not treated explicitly

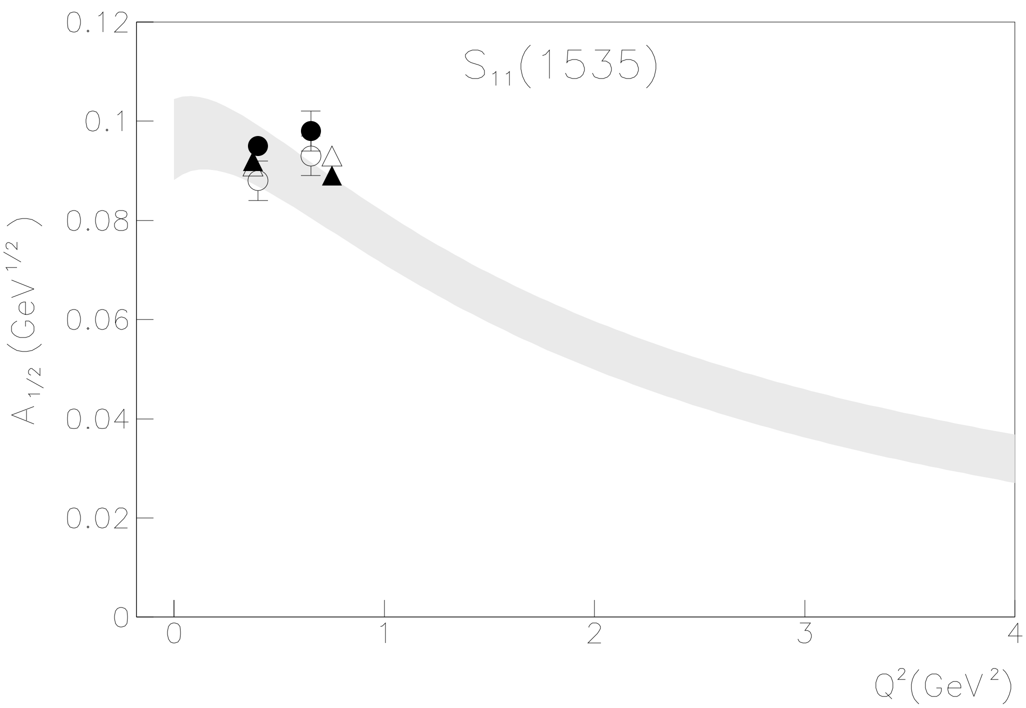
[$\gamma N \rightarrow (\pi\Delta, \rho N \cdot \cdot) \rightarrow \pi N$ is neglecte]

$\rightarrow$

The extracted N^* parameters need to be verified

Jlab/Yerevan UIM global fits to $p\pi^0$, $n\pi^+$, and $p\eta$

$\rightarrow$ Obtain highly constrained $\gamma N \rightarrow N^*$ form factors



More will be given by I. Aznauryan's talk

Recent results from MAID :

$\gamma N \rightarrow N^*$ form factors have been extracted
(L. Tiator's talk)

- Multi-channel K-matrix models
  - SAID :

    Consider γN, πN, $\pi\Delta$(all inelastic channels)

    $\rightarrow$

$$T_{\gamma N,\pi N}(SAID) = A_I(1 + iT_{\pi N,\pi N}) + A_R T_{\pi N,\pi N}$$

$$A_I = K_{\gamma N,\pi N} - \frac{K_{\gamma N,\pi\Delta} K_{\pi N,\pi N}}{K_{\pi N,\pi\Delta}}$$

$$A_R = \frac{K_{\gamma N,\pi\Delta}}{K_{\pi N,\pi\Delta}}$$

Actual analysis:

$$A_I = v^{bg}_{\gamma N,\pi N} + \sum_{n=0}^{M} \bar{p}_n z Q_{l_\alpha+n}(z)$$

$$A_R = \frac{m_\pi}{k_0}\left(\frac{q_0}{k_0}\right)^{l_\alpha} \sum_{n=0}^{N} p_n \left(\frac{E_\pi}{m_\pi}\right)^n$$

$\bar{p}_n, p_n$: fitting parameters

N^* parameters are extracted by fitting the resulting amplitudes to a Briet-Wigner parameterization at $W \rightarrow M^*$

Results from SAID :

1. Determine $\pi N \rightarrow \pi N$, $\gamma N \rightarrow \pi N$ amplitudes
2. extract N^* parameters

Comments : :

Coupled-channel effects are not treated explicitly

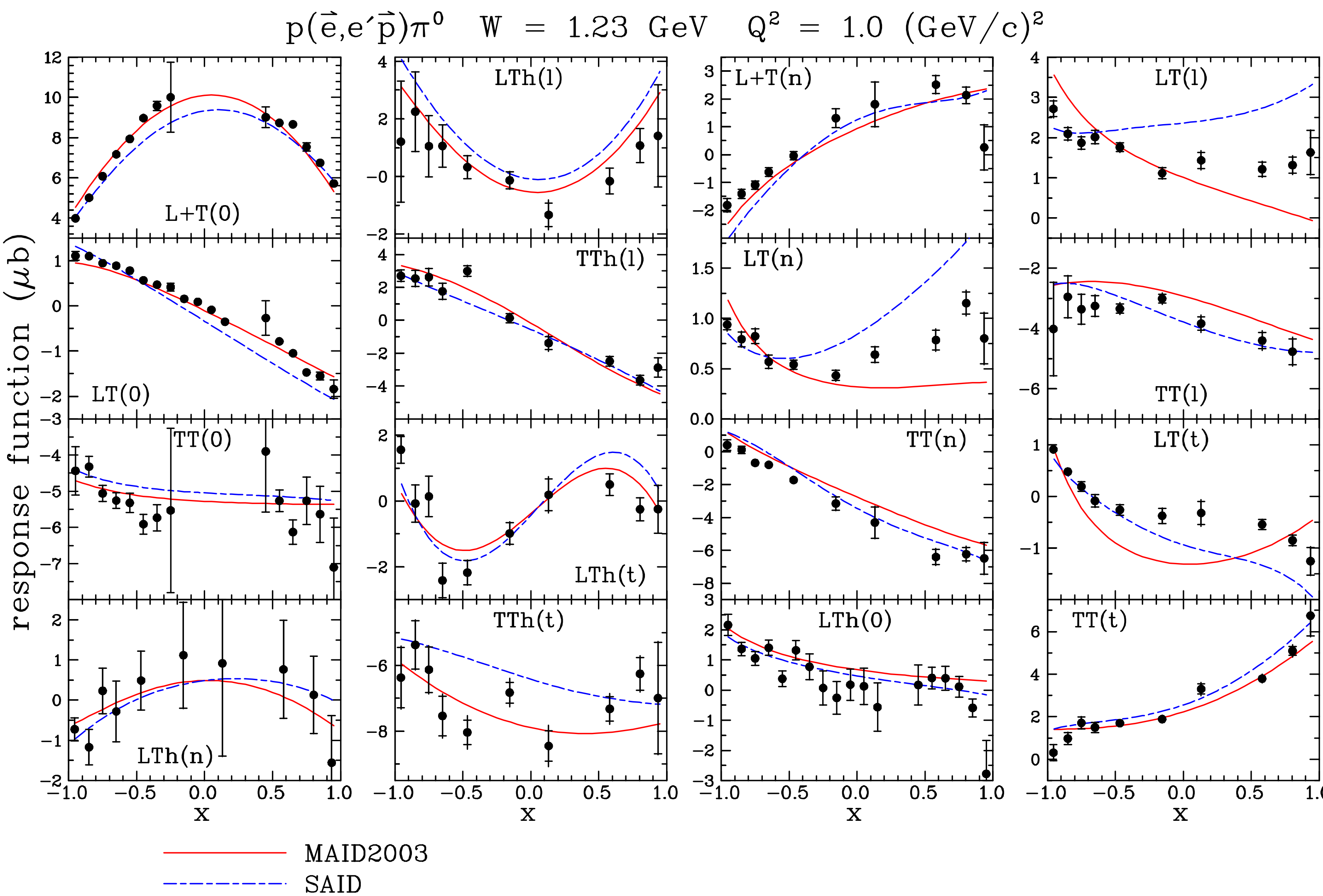
[$\gamma N \rightarrow (\pi\Delta, \rho N \cdot \cdot) \rightarrow \pi N$ is neglected]

$\rightarrow$

The extracted N^* parameters need to be verified

SAID (2005)

$$T_{\gamma N,\pi N}(SAID) = A_I(1 + iT_{\pi N,\pi N}) + A_R T_{\pi N,\pi N} + (C + iD)(Im T_{\pi N,\pi N} - T^2_{\pi N,\pi N})$$



Data :

16 response functions (J. Kelly et al. (2005))

– Giessen Model and KVI Model :

Set $K \rightarrow V$ = Tree-daigrams

$\rightarrow$

$$T_{a,b}(E) = \sum_{c}[(1 + iV(E))^{-1}]_{a,c}V_{c,b}(E)$$

Comments :

Higher-order and off-shell effects are neglected
(Note : $K = V + V\frac{P}{E-H_0}K$)

$\rightarrow$
The extracted N^* parameters can not be interprted in terms of quark models and/or LQCD
(Lesson from the study of Δ)

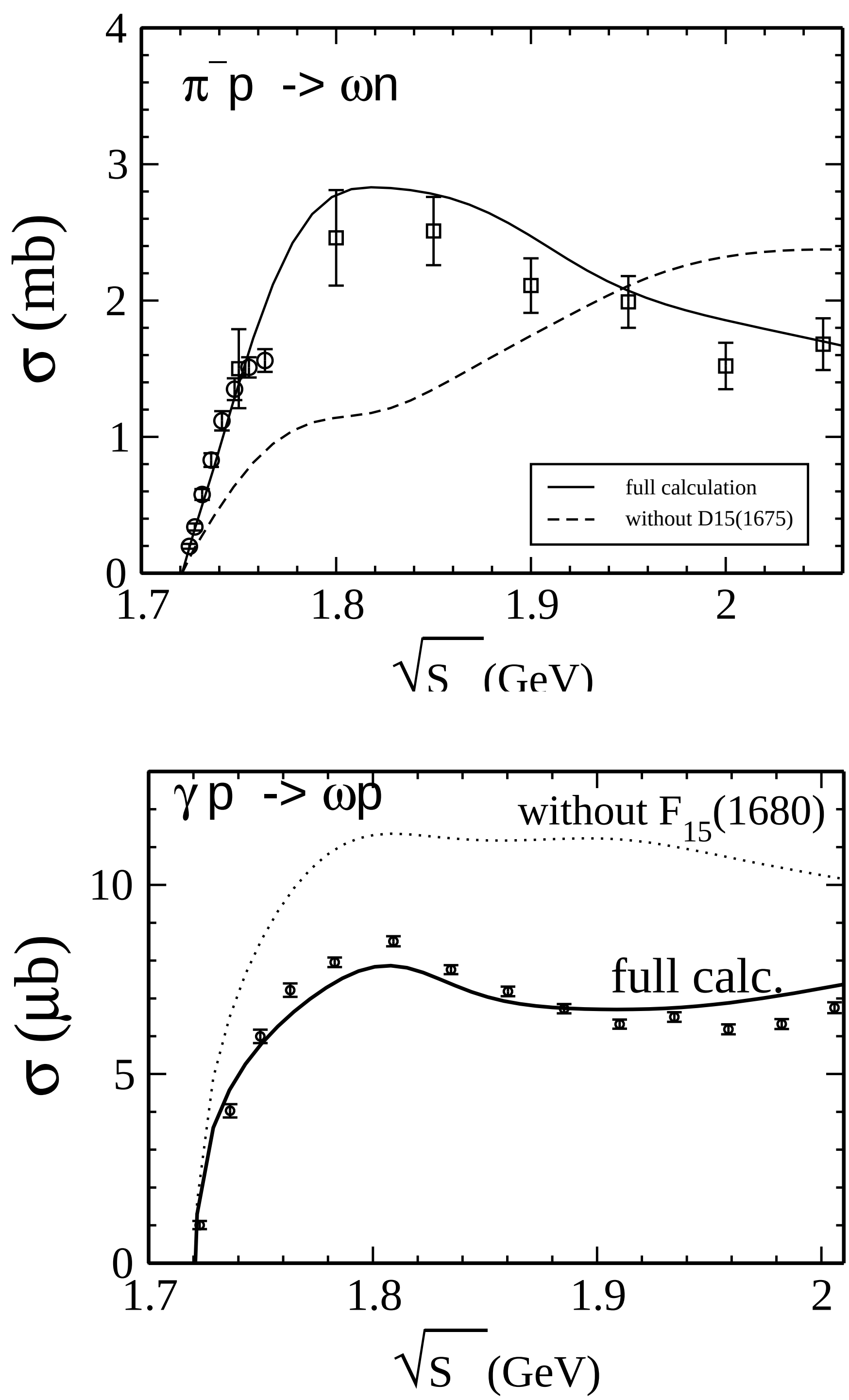
Recent results of Giessen Model
(V. Shklyar et al. (2005)):

1. Include γN, πN, $2\pi N$, ηN, and ωN.

2. Find evidence for $D_{15}(1675)$ in $\pi N \to \omega N$

3. Find evidence for $(F_{15}(1680)$ in $\gamma N \to \omega N$

Recent results from Giessen Model (2005)



Recent Results of KVI Model

(A. Usov and O. Scholten (2005))

– Include πN, γN, $K\Lambda$, $K\Sigma$, ϕN, ηN

– Fit the data of $\gamma p \to K^+\Lambda, K^+\Sigma^0, K^0\Sigma^+$
(Show large coupled-channel effects)

Comments :

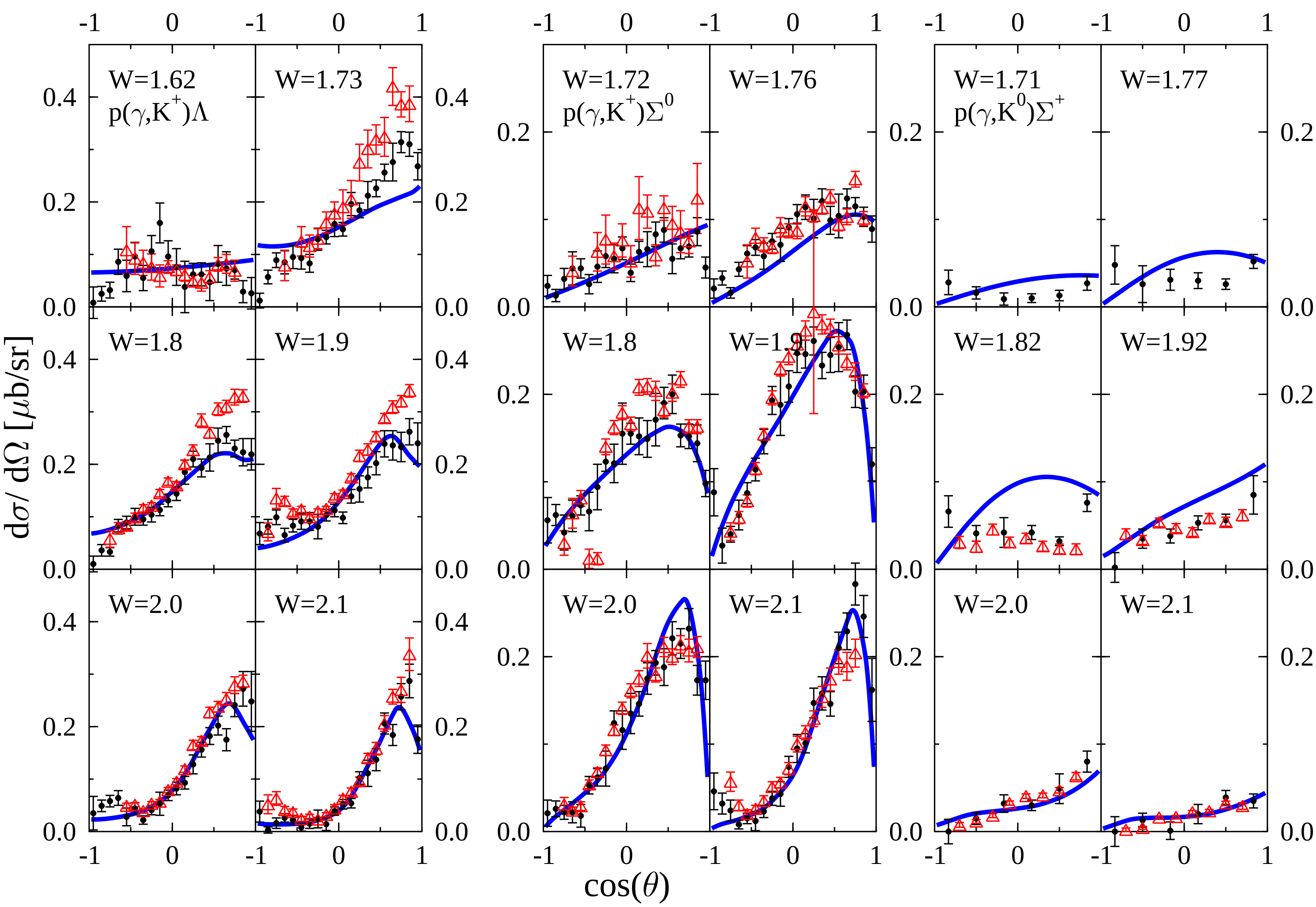
$2\pi N$ channel is not included

(Note : $\sigma_{\gamma N \to \pi\pi N} \gg \sigma_{\gamma N \to KY}$)

$\to$

The extracted N^* parameters need to be verified

$\gamma p \to KY$ results of KVI Model (2005)



More will be given in O. Scholten's talk

For deriving:

- Carnegie-Mellon Berkeley (CMB) Model
- Kent State University (KSU) model
- Dynamical models

Apply two-potential scattering formulation

for $V = v^{bg} + \frac{\Gamma^\dagger_{N*}\Gamma_{N*}}{E-M^0_{N*}}$
$\rightarrow$

$$T(E) = t^{bg}(E) + \frac{\bar{\Gamma}^\dagger_{N*}(E)\bar{\Gamma}_{N*}(E)}{E - M^0_{N*} - \Sigma_{N*}(E)}$$

$$t^{bg} = v^{bg} + v^{bg}G(E)t^{bg}(E)$$

Resonance parameters :

$$\bar{\Gamma}_{N*} = \Gamma_{N*} + \Gamma_{N*}G(E)t^{bg}(E)$$

$$\Sigma_{N*}(E) = \Gamma^\dagger_{N*}G(E)\bar{\Gamma}_{N*}$$

Main features :

- Isolate resonant term $\sim$ Briet-Wigner form
- Non-resonant effects on resonance parameters are identified

For multi -channel multi -resonant case:

$$T_{a,b}(E) = t^{bg}_{a,b}(E) + \sum_{N^*_i,N^*_j} \bar{\Gamma}^\dagger_{N^*_i,a}(E)[\hat{G}(E)]_{i,j}\bar{\Gamma}_{N^*_j,b}(E)$$

$$t^{bg}_{a,b} = v^{bg}_{a,b} + \sum_c v^{bg}_{a,c} G_c(E) t^{bg}_{c,b}(E)$$

$$\bar{\Gamma}_{N^*,a} = \Gamma_{N^*,a} + \sum_b \Gamma_{N^*,b} G_b(E) t^{bg}_{b,a}$$

$$[\hat{G}(E)^{-1}]_{i,j}(E) = (E - M^0_{N^*_i})\delta_{i,j} - \Sigma_{i,j}(E)$$

$$\Sigma_{i,j}(E) = \sum_a \Gamma^\dagger_{N^*,a} G_a(E) \bar{\Gamma}_{N^*_j,a}$$

a,b $= \pi N$, γN, ηN, ωN, KY, σN, ρN, $\pi\Delta$ $(\pi\pi N)$

- Carnegie-Mellon Berkeley (CMB) Model

Set : $v^{bg}_{a,b}(E) = \frac{\Gamma^\dagger_{L,a}\Gamma_{L,b}}{E-M_L} + \frac{\Gamma^\dagger_{H,a}\Gamma_{H,b}}{E-M_H}$

$\rightarrow$

$$V = v^{bg} + v^R = \sum_{i=N^*_i,L,H} \frac{\Gamma^\dagger_{i,a}\Gamma_{i,b}}{E-M_i} = Separable$$

$\rightarrow$

$$T_{a,b}(E) = \sum_{i,j} \Gamma^\dagger_{i,a} G_{i,j}(E) \Gamma_{j,b}$$
$$G(E)^{-1}_{i,j} = (E - M^0_i)\delta_{i,j} - \Sigma_{i,j}(E)$$
$$\Sigma_{i,j}(E) = \sum_a \int k^2 dk \frac{\Gamma^\dagger_{i,a}(k)\Gamma_{j,a}(k)}{E - E_{M_a}(k) - E_{B_a}(k) + i\epsilon}$$

With appropriate variable changes : $s = E^2$

$\rightarrow$

CMB's dispersion relations :

$$\Sigma_{i,j}(s) = \sum_c \gamma_{i,c}\Phi_c(s)\gamma_{j,c}$$
$$Re[\Phi_c(s)] = Re[\Phi_c(s_0)] + \frac{s - s_{th,c}}{\pi}\int_{s_{th}}^{\infty} \frac{Im[\Phi_c(s')]}{(s'-s)(s'-s_0)} ds'$$

$\rightarrow$

CMB model is analytic

Recent applications/extensions of CMB model :

– Zagreb : M. Batinic, A. Svarc and collaborators

Consider three channels : πN, ηN, $\sigma(\pi\pi)N$

– PITT-ANL : T. Varana, S. Dytman, T.-S. H. Lee

Consider up to eight channels:
πN, ηN, $\pi\Delta$, ρN, $\sigma(\pi\pi)N$, $\pi N^*(1440)$, $K\Lambda$, γN

– FSU-PITT :
A. Kiswandhi, S.Capstick, and S. Dytman

Investigate model-dependence in S_{11} channel

Results:

- N^* in S_{11} channel is better understood
- The interplay between channel coupling and N^* excitation has been better understood
- Some extracted N^* parameters are significantly different from PDG values

Current effort (A. Kiswandhi, S. Capstick) :

Approach is being developed to replace

$v^{bg}_{a,b}(E) = \frac{\Gamma^\dagger_{L,a}\Gamma_{L,b}}{E-M_L} + \frac{\Gamma^\dagger_{H,a}\Gamma_{H,b}}{E-M_H}$

by dynamical models

- Kent State University (KSU) model

Start with

$$T(E) \ = \ t^{bg}(E) + \frac{\bar{\Gamma}^{\dagger}_{N*}\bar{\Gamma}_{N*}}{E - M^0_{N*} - \Sigma_{N*}(E)}$$

One can derive exactly the distorted-wave form

$$\begin{aligned} S(E) \ &= \ 1 + 2iT(E) \\ &= \ \omega^{(+)T} R(E) \omega^{(+)} \end{aligned}$$

where

$$\begin{aligned} \omega^{(+)} \ &= \ 1 + G(E)t^{bg}(E) \\ R(E) \ &= \ 1 + 2iT^R(E) \\ T^R(E) \ &= \ \frac{\Gamma^{\dagger}_{N*}(E)\Gamma_{N*}(E)}{E - M^0_{N*} - \Sigma_{N*}(E)} \end{aligned}$$

KSU separable parameterization:

$$T^R(E) \ = \ \frac{K}{1 + iK}$$

$$K_{ij} \ = \ \sum_{\alpha} tan\delta_{\alpha} f_{i\alpha} f_{j\alpha}$$

$$\omega^{(+)} \ = \ B_1 B_2 \cdot \cdot B_n$$

$$B_i \ \sim \ e^{iX\Delta_i}$$

Recent Results from KSU Model

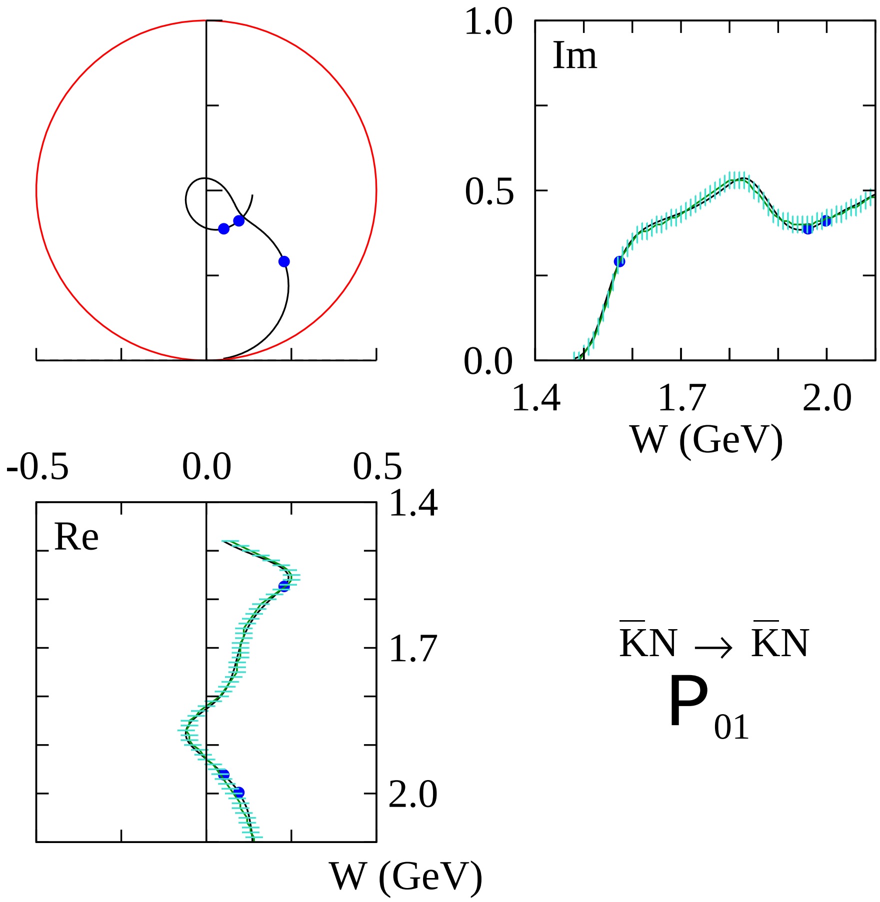
- Fit $S = -1$ amplitudes with Channels :
  $\bar{K}n, \pi\Lambda, \pi\Sigma, \pi\Sigma^*(1385), \pi\Sigma^*(1520), \bar{K}\Delta, \bar{K}^*N, \eta N$

  $\rightarrow$

  Extract Λ^* and Σ^* in $W = 1560 - 1685$ MeV

- Analyze data of Crystal Ball Collaboration
  $K^-p \rightarrow$ neutrals ($\bar{K}^0 n, \pi^0\Lambda, \pi^0\Sigma^0 \cdots$)

KSU fits to $\bar{K}N \to \bar{K}N$ (2005)



Dynamical Models

Two equivalent approaches:

- Solve dynamical equations with $V = v^{bg} + v^{R}$ directly :

$$T_{a,b}(E) \; = \; V_{a,b} + \sum_c V_{a,c} G_c(E) T_{c,b}(E)$$

$a, b, c = \pi N, \gamma N, \eta N, \pi\Delta \cdots$

Recent works :

– Juelich Model :πN
– Fuda et al. :πN, γN
– DMT Model : πN, γN, ηN
– Ohio-Utrecht Model : πN, γN
– Chiral SU(3) models : KY, ωN, γN, πN
(set v^{R} =0)

- Use two-potential formulation to identify resonant mechanism

$$
\begin{aligned}
T_{a,b}(E) &= t^{bg}_{a,b}(E) + \sum_{N^*_i,N^*_j} \bar{\Gamma}^{\dagger}_{N^*_i,a}[D^{-1}(E)]_{i,j}\bar{\Gamma}_{N^*_j,b} \\
t^{bg}_{a,b}(E) &= v^{bg}_{a,b} + \sum_c v^{bg}_{a,c} G_c(E) t^{bg}_{c,b}(E) \\
\bar{\Gamma}_{N^*,a} &= \Gamma_{N^*,a} + \sum_b \Gamma_{N^*,b} G_b(E) t^{bg}_{b,a}(E)
\end{aligned}
$$

Recent Works

- Sato-Lee Model : πN, γN
- Yoshimoto et al. : πN, ηN, $\pi\Delta$
- Julia-Diaz et al. : γN, KY, πN
- Matsuyama, Lee, Sato : γN, πN, ηN, ωN, $\pi\pi N$

Juelich's Coupled-channel Model

O. Krehl, C. Hanhart, S. Krewald, J. Speth (2000)

- Channels : $\pi N, \eta N, \sigma N, \pi \Delta, \rho N$.
- Main result:
  P_{11} is due to meson-baryon coupled-channel effects

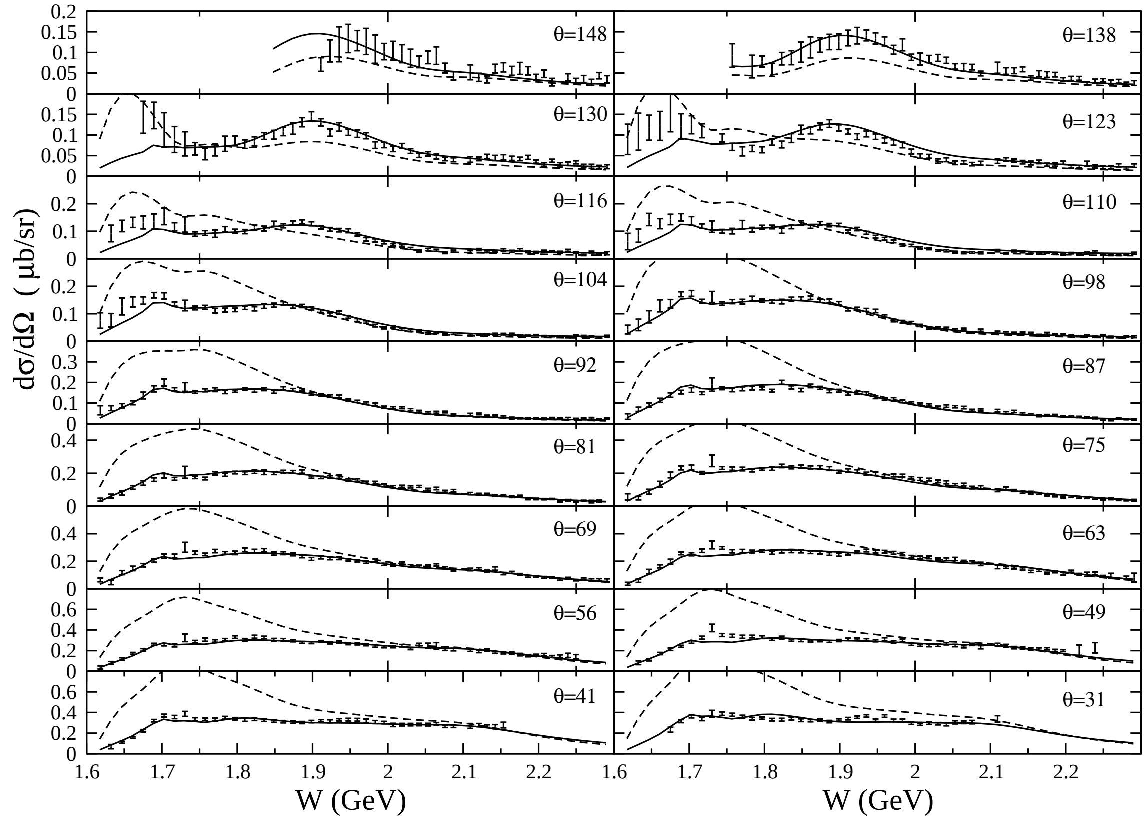
Being revised and extended to investigate $\gamma N \to \pi N$
(K. Nakayama et al. (2005))

Coupled-channel Model for KY production

B. Julia-Diaz, B.Saghai, F. Tabakin, T.-S. H. Lee (2005)

- Channels : γN, πN, $K\Lambda$, $K\Sigma$
- fit SAPHIR and JLAB data of $\gamma N \rightarrow K^+\Lambda$
- Main Result :

  Large coupled-channel effects due to πN channel



Recent Results of B. Julia-Diaz et al. (2005)

More will be presented in T.-S. H Lee's talk

DMT Coupled-channel Model

C.-Y Chen, S. Kamalov, S.N. Yang, D. Drechsel, L. Tiator

- Channels : γN, $\pi N, \eta N$
- $\pi\pi N$ effects are assumed to be in $\Gamma_{N^*} = \Gamma_{1\pi} + \Gamma_{2\pi}$
- Main Results :

  Need 4 N^* to fit S_{11} amplitudes

  (show meson cloud effects on $\gamma N \to S_{11}(1535)$)

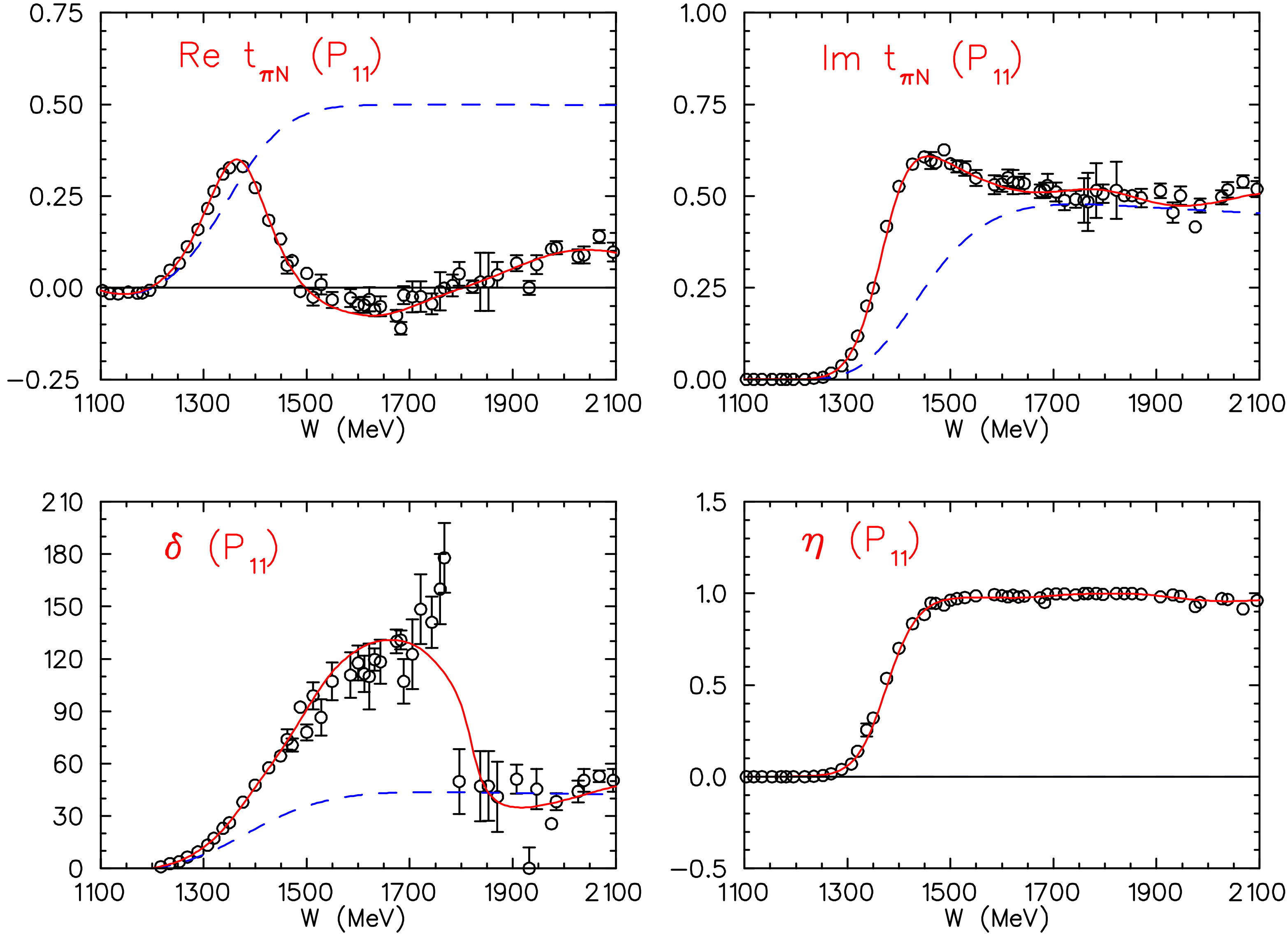
  Fit to all partial waves will soon be completed

Comments :

Need to justify its simple treatment of $\pi\pi N$ channels

(Note : $\sigma_{2\pi N} > \sigma_{\pi N}$ at $W > 1.5$ GeV)

Recent Results from DMT Model



Coupled-channel Model with $\pi\pi N$ channel

A. Matsuyama, T. Sato , T.-S. H. Lee (in progress)

- Channels : γN, πN, ηN, ωN, $\pi\pi N$ ($\pi\Delta$, ρN, σN)
- Apply second order unitary transformation on H
- Satisfy $\pi\pi N$ unitarity condition :

$$Im f_{a,a}(\theta = 0) = \sum_b \sigma_{a,b} + \sigma_{a,\pi\pi N}$$

$a, b = \pi N$, γN, ηN, ωN (stable particle channels)

$\rightarrow$

Coupled-channel equation with $\pi\pi N$ cut

Coupled-channel equation with $\pi\pi N$ cut :

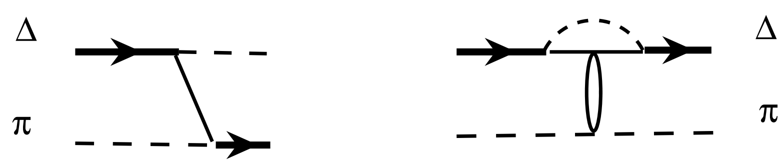
$$X_{a,b}(E) \;=\; Z_{a,b}(E) + \sum_c Z_{a,c}(E) G_c(E) X_{cb}(E)$$

$$a, b \;=\; \gamma N, \pi N, \eta N, \omega N, \pi\Delta, \rho N, \sigma N$$

$Z(E) = v^{bg} + Z^{cut}(E)$

v^{bg} = (tree-diagrams of Chiral Lagrangians)

$Z^{cut}(E)$:



Apply Spline function method :

- solve coupled-channel equations with $\pi\pi N$ cut
- include $\pi\pi N$ cut effects exactly to calculate

  $\pi N \rightarrow \pi\pi N$

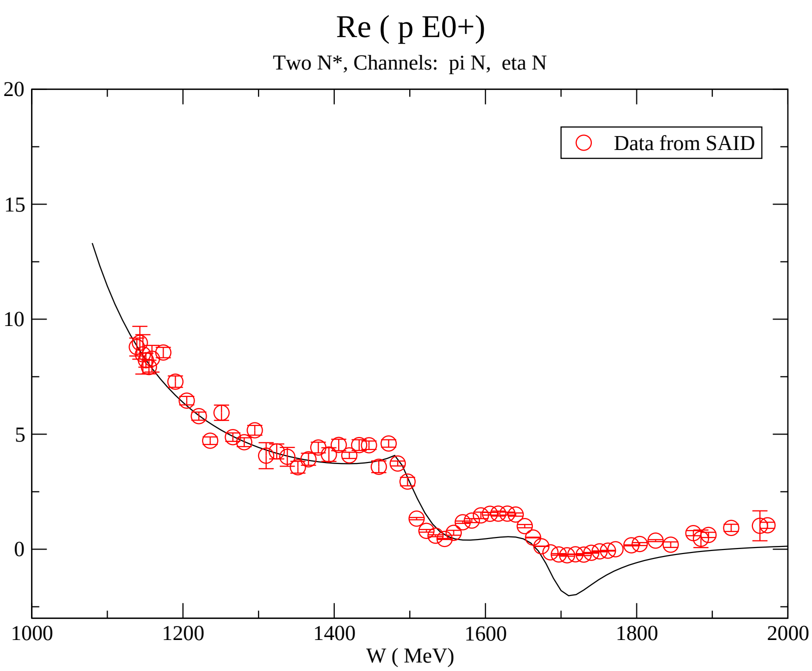
  $\gamma N \rightarrow \pi\pi N$

  (Note : can not be achieved by contour rotation)

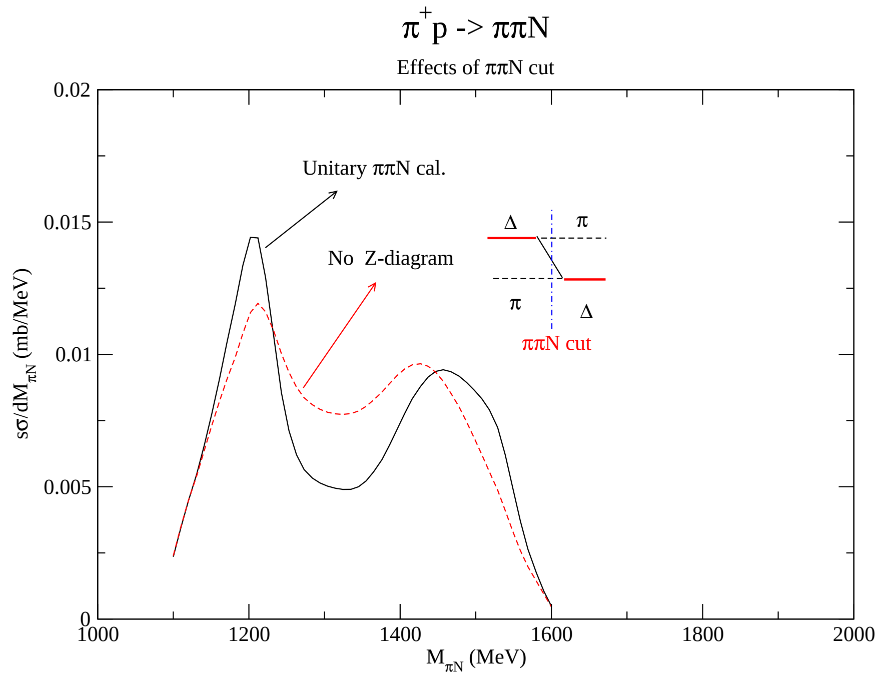
- Main Results (2005) :
  - $\gamma N \rightarrow S_{11}(1535)$:
    1. Meson cloud effect is about 20%
    2. Bare helicity amplitude is close to quark model
  - $\pi\pi N$ unitary cut is crucial in predicting $\pi\pi N$ production cross sections

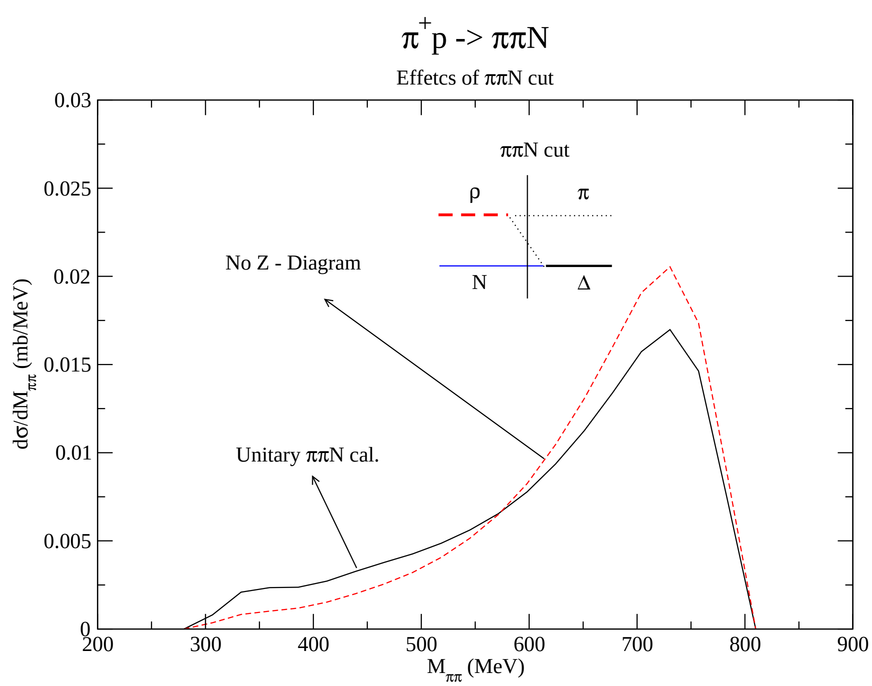
	$\gamma N \rightarrow S_{11}(1535)$	
$\pi\pi N$ model	Dressed	61.24
	Bare	77.64
Capstick		76

$$\bar{\Gamma}_{\gamma N,N^*} = \underbrace{\Gamma_{\gamma N,N^*}}_{Bare} + \sum_{MB=\pi N,\eta N,\pi\Delta} v^{bg}_{\gamma N,MB} G_{MB} \bar{\Gamma}_{MB,N^*}$$



Effect of $\pi\pi N$ cut





If coupled-channel and $\pi\pi N$ cut are neglected

$\rightarrow$

- Moscow/JLab Isobar Model of $\gamma N \rightarrow \pi\pi N$
  (V. Mokeev's talk)

- Amplitude analyses of $\gamma N \rightarrow \pi\pi N$
  of RPI/JLab

Chiral SU(3) coupled-channel models

N. Kaiser, E. Oset, A. Ramos, U. Messiner $\cdots$
M. Lutz, E. Kolomeitsev

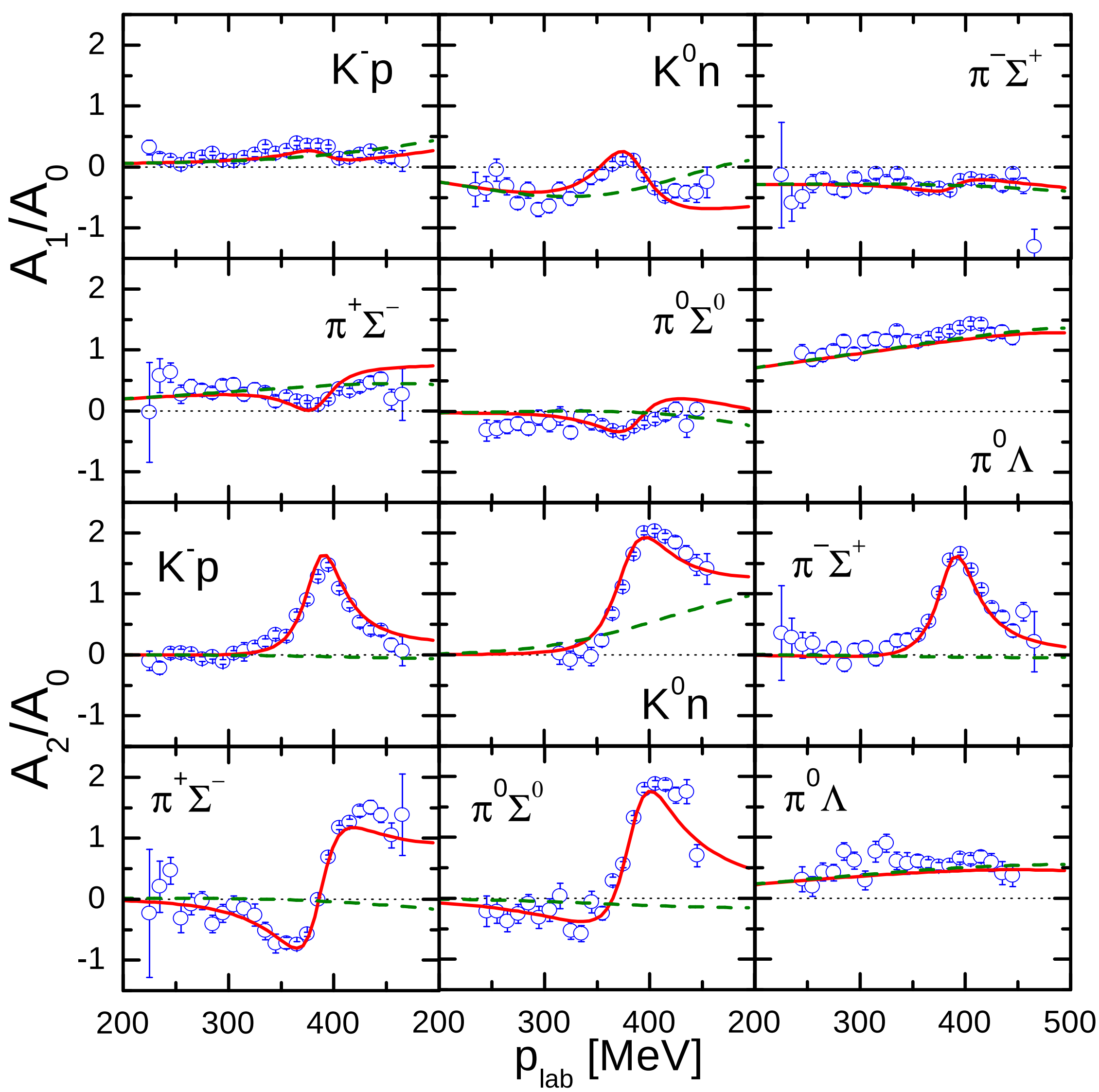
- no N^* field is included explicitly
- V : SU(3) chiral Lagrangians up to $(Q/\Lambda)^n)$
- Numerical strategy :
  - On-shell factorization (N/D method)
  - chiral counting : u-channel $V \to$ separable form

  $\to$

  BS Equations $\to$ separable $\to$ algebraic equations
- Recent Results :
  - M. Lutz, E. Kolomeitsev :
    Can fit both πN and KN data (22 parameters)
  - T. Inoue, E. Oset et al. :
    Study $\pi N \to \pi\pi N$ near threshold
  - $\cdot\cdot\cdot\cdot\cdot$

Results of M. Lutz, E. Kolomeitsev :

$$\frac{d\sigma}{dcos\theta} = A_0 + A_1 P_1(cos\theta) + A_2 P_2(cos\theta)$$



More will be given in M. Lutz's talk

Concluding Remarks

- Amplitude Analyses and Dynamical Reaction Models are complementary in N^* program

  - Has been realized in Δ region
  - to be developed in the 2nd and 3rd N^* regions

- Gauge invariance is problematic

  Due to the need of using phenomenlogical form factors and/or regularization constants

  (will be discussed in Haberzettl's talk)

- Coupled-channel approach is mandetory for analyzing data of weak channels (ηN, KY, ωN)

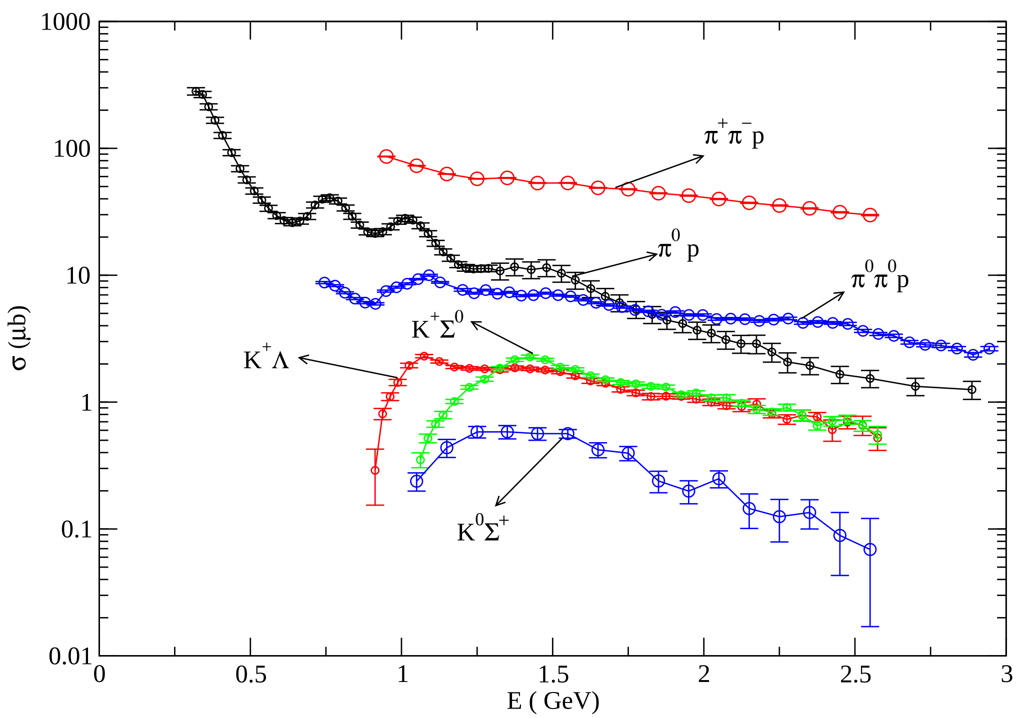
  Example : $\gamma p \to K\Lambda$

  Unitarity Condition ($i[T - T^+] = TT^+$)

$$Im[T_{\gamma p, K\Lambda}] = \sum_{MB} T_{\gamma p, MB} T^*_{K\Lambda, MB}$$

$$\propto \sum_{MB} \sqrt{\sigma_{\gamma p, MB}} \sqrt{\sigma_{K\Lambda, MB}}$$

γp Reaction Cross Sections



$\rightarrow$

$\gamma p \to (\pi N, \pi\pi N) \to K^+\Lambda$ must be significant

- collaborations are important